IMPLEMENTING THE STANDARDS FOR MATHEMATICAL PRACTICE IN THE HIGH SCHOOL CLASSROOM

MELANIE JANZEN, SAN BERNARDINO COUNTY SUPERINTENDENT OF SCHOOLS

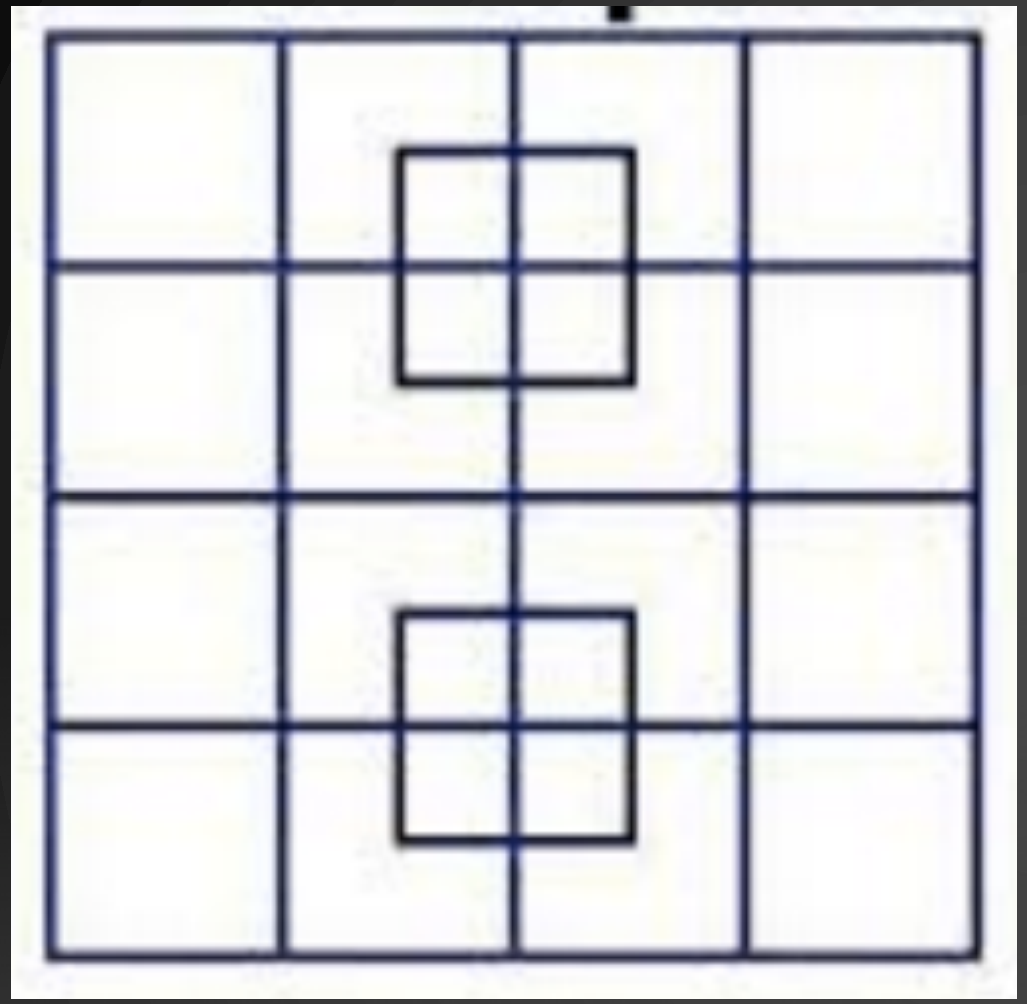
CATHERINE VITTORIO, SAN BERNARDINO COUNTY SUPERINTENDENT OF SCHOOLS

SAVANNAH SPRAGUE, TEACHER OF TOMORROW

Yesterday evening, Helen and her husband invited their neighbors (two couples) for a dinner at home. The six of them sat at a round table. Helen tells you the following:

- “Victor sat on the left of the woman who sat on the left of the man who sat on the left of Anna.”
- Esther sat on the left of the man who sat on the left of the woman who sat on the left of the man who sat on the left of the woman who sat on the left on my husband.
- Jim sat on the left of the woman who sat on the left of Roger.
- I did not sit beside my husband.

The Question: What is the name of Helen’s husband?

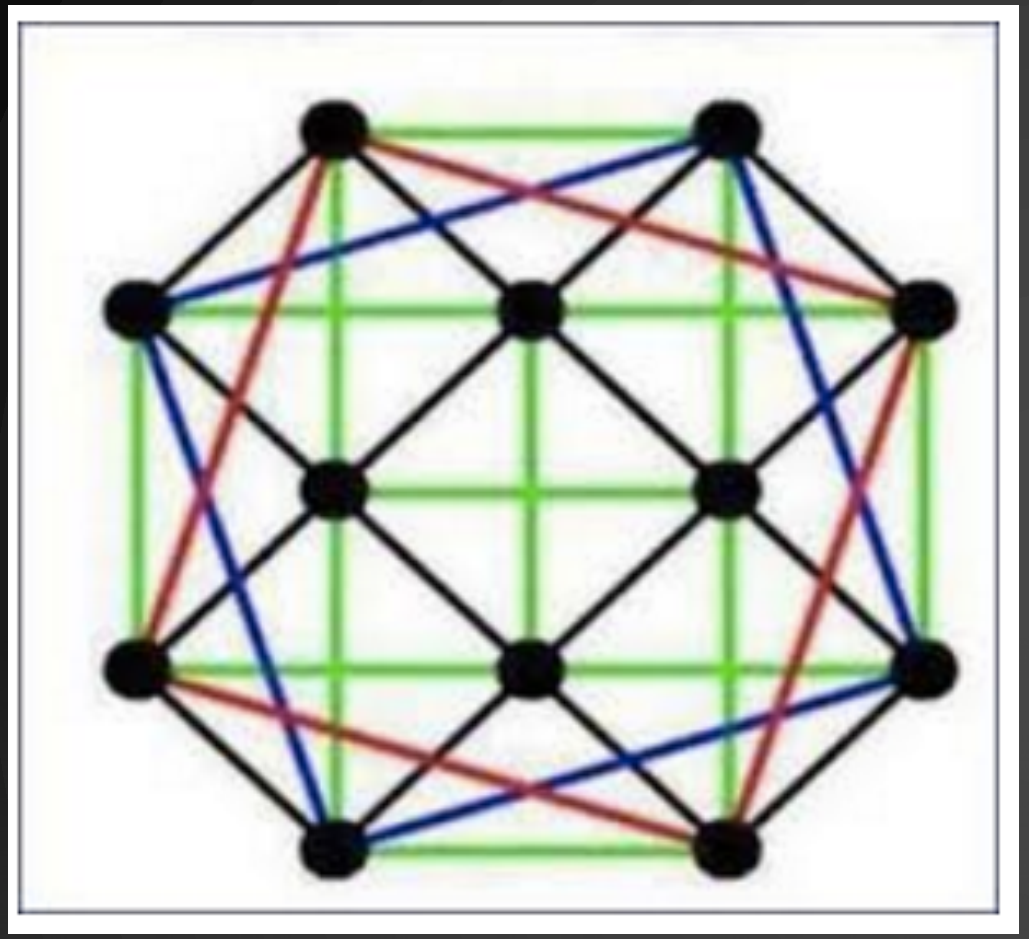


What do you notice?

What do you wonder?

How many squares can you identify?

How did you approach the problem?



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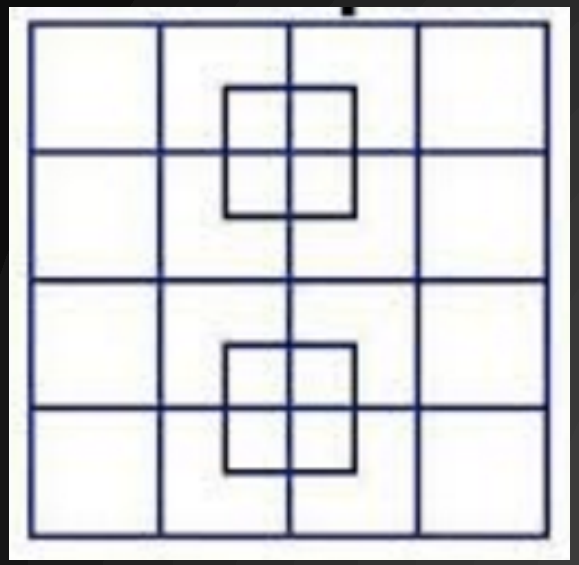
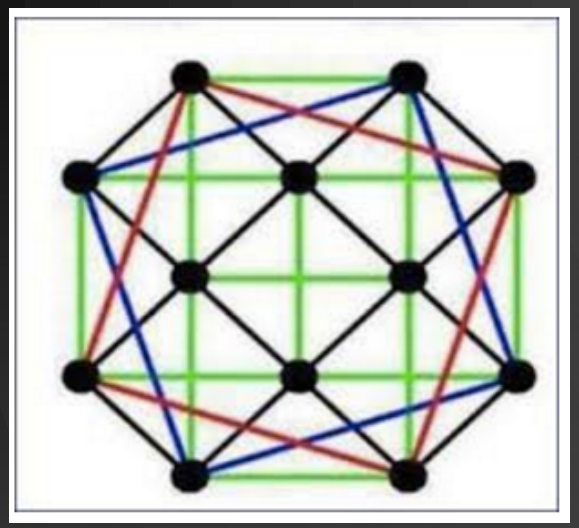




What similarities do you find between the two?

What differences do you find between the two?

FILL IN THE GRID SO THAT EACH COLUMN, ROW AND DIAGONAL ADD UP TO THE SUM.

Sum: 15

3		7
4		

What do you notice?

What do you wonder?

How would you go about solving this?

FILL IN THE GRID SO THAT EACH COLUMN, ROW AND DIAGONAL ADD UP TO THE SUM.

Sum: 15

3		7
4		

Sum: 45

21		9
		24

FILL IN THE GRID SO THAT EACH COLUMN, ROW AND DIAGONAL ADD UP TO THE SUM.

Now, create your own!

Where should you start?

Sum: ??

WHAT WAS THE POINT?

Using the Standards for Mathematical Practice in a content free context.

WHAT'S THE BIG DEAL?

- The eight Standards for Mathematical Practice (MP) describe the **attributes** of mathematically proficient students and expertise that mathematics educators at all levels should seek to develop in their students.
- Mathematical practices provide a vehicle **through which students engage with and learn mathematics**. As students move from elementary school through high school, mathematical practices are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

- Taken from the California Mathematics Framework, 2013

These practices rest on important **"processes and proficiencies"** with longstanding importance in mathematics education.

1. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections.
2. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010q).

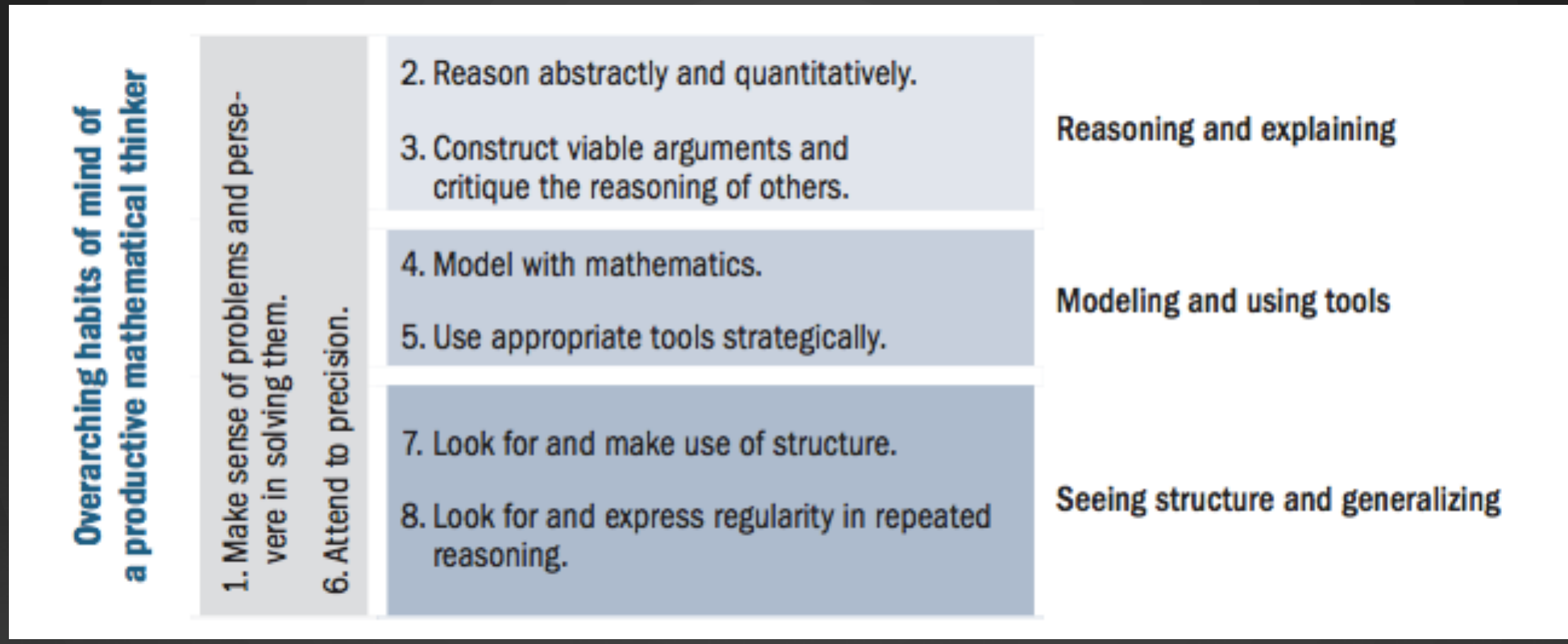
Taken from the California Mathematics Framework, 2013

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

What Standards for Mathematical Practice have we used so far?

4 CATEGORIES OF SMPs



WHAT IMPLICATIONS DOES THIS HAVE FOR ME IN MY CLASSROOM?

- Working on students habits of mind
- Preparing for success in college level mathematics
- Assessed on the SBAC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4 CLAIMS OF THE SBAC

Claim #1: Concepts and Procedures
Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Claim #2: Problem Solving
Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

Claim #3: Communicating Reasoning
Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Claim #4: Modeling and Data Analysis
Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems."

CLAIM #1: CONCEPTS AND PROCEDURES

- Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Number and Quantity (9-12.N)
Target A [a/s]: Extend the properties of exponents to rational exponents. (DOK 1, 2)
Target B [a/s]: Use properties of rational and irrational numbers. (DOK 1, 2)
Target C [m]: Reason quantitatively and use units to solve problems. (DOK 1, 2)
Algebra (9-12.A)
Target D [m]: Interpret the structure of expressions. (DOK 1)
Target E [m]: Write expressions in equivalent forms to solve problems. (DOK 1, 2)
Target F [a/s]: Perform arithmetic operations on polynomials. (DOK 1)
Target G [a/s]: Create equations that describe numbers or relationships. (DOK 1, 2)
Target H [m]: Understand solving equations as a process of reasoning and explain the reasoning. (DOK 1, 2)
Target I [m]: Solve equations and inequalities in one variable. (DOK 1, 2)
Target J [m]: Represent and solve equations and inequalities graphically. (DOK 1, 2)
Functions (9-12.F)
Target K [m]: Understand the concept of a function and use function notation. (DOK 1)
Target L [m]: Interpret functions that arise in applications in terms of a context. (DOK 1, 2)
Target M [m]: Analyze functions using different representations. (DOK 1, 2, 3)
Target N [m]: Build a function that models a relationship between two quantities. (DOK 1, 2)
Geometry (9-12.G)
Target O: Define trigonometric ratios and solve problems involving right triangles (DOK 1, 2)
Statistics and Probability (9-12.SP)
Target P [m]: Summarize, represent and interpret data on a single count or measurement variable. (DOK 2)

CLAIM #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

SMP #1 – Make sense of problems and persevere in solving them.

SMP #5 – Use appropriate tools strategically.

SMP #7 – Look for and make use of structure.

SMP #8 – Look for and express regularity in repeated reasoning.

CONTENT STANDARDS ASSOCIATED WITH CLAIM #2 PROBLEM SOLVING

- N-Q.A
- A-SSE.A
- A-SSE.B
- A-CED.A
- A-REI.2
- A-REI.B
- A-REI.C
- A-REI.D
- F-IF.A
- F-IF.B
- F-IF.C
- F-BF.A
- G-SRT.C
- S-ID.C
- S-CP.A

SAMPLE TASK FOR CLAIM #2

"Toys for Charity" **(First-year Algebra)**

Phil and Cathy want to raise money for charity. They decide to make and sell wooden toys.
They could make them in two sizes: small and large.

Phil will carve them from wood. A small toy takes 2 hours to carve and a large toy takes 3 hours to carve.
Phil only has a total of 24 hours available for carving.

Cath will decorate them. She only has time to decorate 10 toys.

The small toy will make $8 for charity.
The large toy will make $10 for charity.

They want to make as much money for charity as they can.

How many small and large toys should they make?

How much money will they then make for charity?

A-REI.C

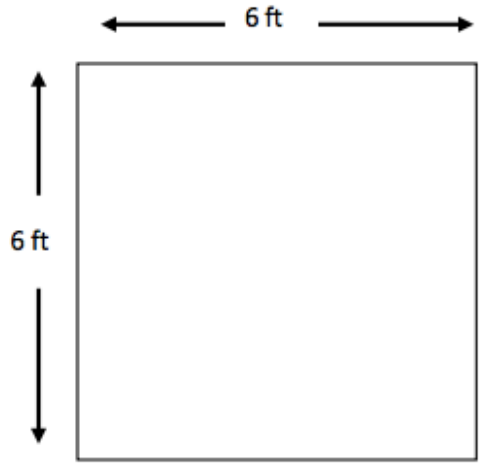
Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

- **Solve systems of equations.** [Linear-linear and linear-quadratic]

SAMPLE EXTENDED RESPONSE FOR CLAIM #2

Making a Water Tank **(Grade 11 – students provided graphing calculator as a tool)**

A square metal sheet (6 feet x 6 feet) is to be made into an open-topped water tank by cutting squares from the four corners of the sheet, and bending the four remaining rectangular pieces up, to form the sides of the tank. These edges will then be welded together.



A. How will the final volume of the tank depend upon the size of the squares cut from the corners?

Describe your answer by:

i) Sketching a rough graph

ii) explaining the shape of your graph in words

iii) writing an algebraic formula for the volume

B. How large should the four corners be cut, so that the resulting volume of the tank is as large as possible?

A-CED.A

Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

- **Create equations that describe numbers or relationships.** [Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only]

CLAIM #3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

SMP #3 – Construct viable arguments and critique the reasoning of others.

SMP #6 – Attend to precision.

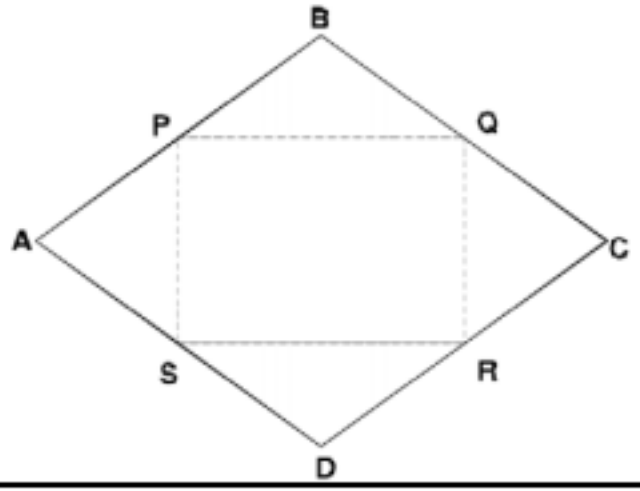
CONTENT STANDARDS ASSOCIATED WITH CLAIM #3 COMMUNICATING REASONING

- N-RN.A
- N-RN.B
- N-RN.3
- A-SSE.2
- A-APR.1
- A-APR.B
- A-APR.4
- A-APR.6
- A-REI.A
- A-REI.1
- A-REI.2
- A-REI.C
- A-REI.10
- A-REI.11
- F-IF.1
- F-IF.5
- F-IF.9
- F-BF.3
- F-BF.4a
- G-CO.A
- G-CO.B
- G-CO.C
- G-CO.9
- G-CO.10
- G-CO.11
- G-SRT.A
- G-SRT.B
- F-TF.1
- F-TF.2
- F-TF.8

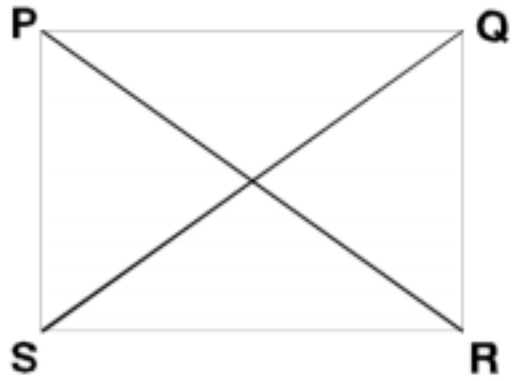
EXAMPLE OF EXTENDED RESPONSE TASK FOR CLAIM #3

The Envelope

Unfolded envelope



Folded envelope



Prove that when the rectangular envelope (PQRS) is unfolded, the shape obtained (ABCD) is a rhombus.

G-CO.11

Claim #3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

- **Prove** theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

CLAIM #4: Students can analyze complex real-world scenarios and can construct and use mathematical model to interpret and solve problems.

SMP #4 – Model with mathematics.

SMP #2 – Reason abstractly and quantitatively.

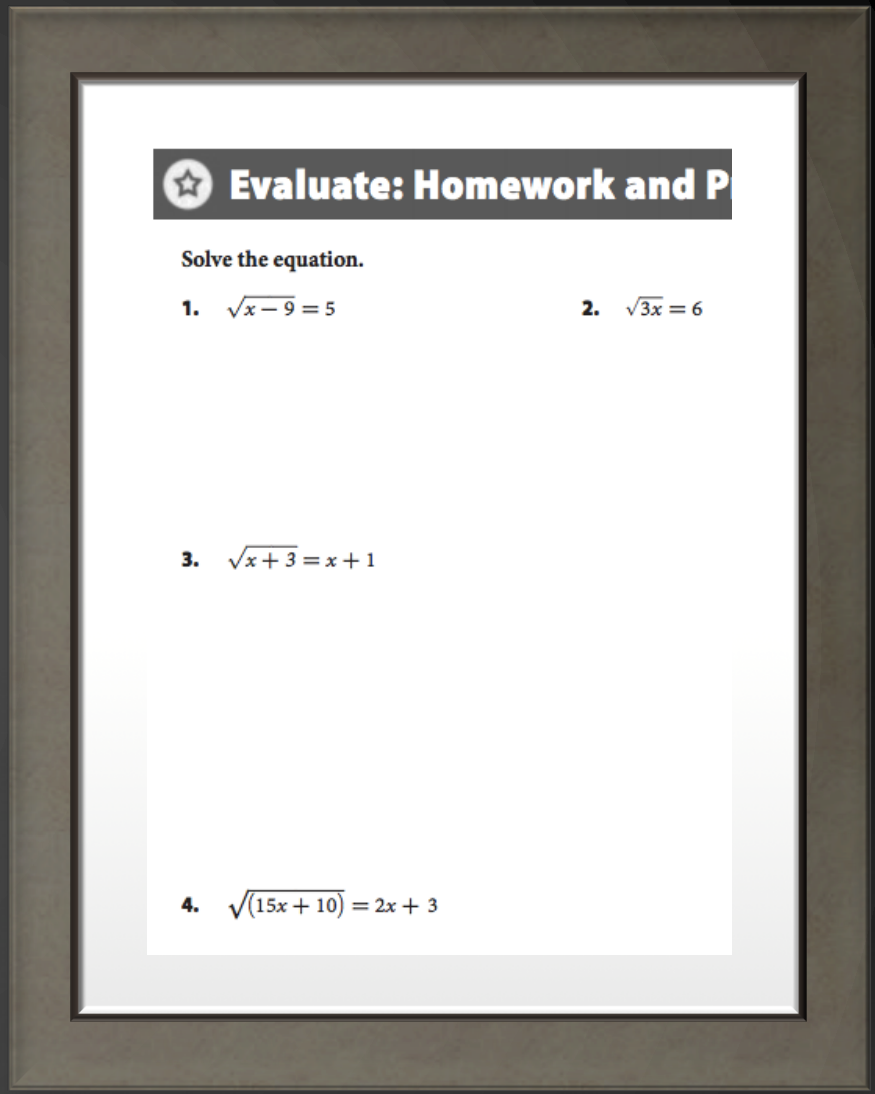
SMP #5 – Use appropriate tools strategically.

CONTENT STANDARDS ASSOCIATED WITH CLAIM #4 MODELING AND DATA ANALYSIS

- N-Q.A
- A-SSE.B
- A-CED.A
- A-REI.A
- A-REI.B
- A-REI.C
- F-IF.B
- F-IF.C
- F-BF.A
- S-ID.A
- S-ID.B
- S-IC.1
- S-IC.B
- F-LE.A
- F-LE.B
- F-TF.5
- G-GMD.3
- G-MG

HOW CAN WE INCORPORATE THE SMPS IN A LESSON?

A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.



Evaluate: Homework and P

Solve the equation.

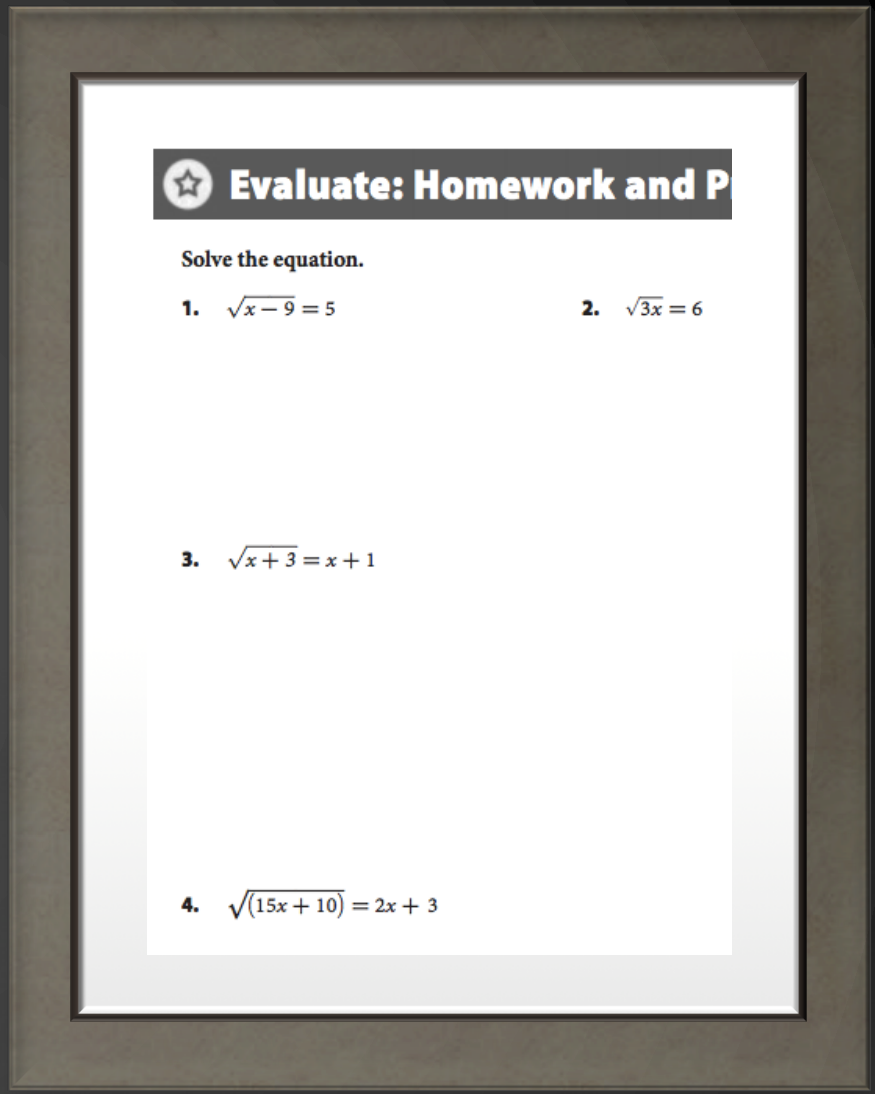
1. $\sqrt{x-9}=5$ **2.** $\sqrt{3x}=6$

3. $\sqrt{x+3}=x+1$

4. $\sqrt{(15x+10)}=2x+3$

HOW CAN WE INCORPORATE THE SMPS IN A LESSON?

A-REI.2 Solve simple rational and radical equations in one variable, ~~and give examples showing how extraneous solutions may arise.~~



Evaluate: Homework and P

Solve the equation.

1. $\sqrt{x-9}=5$

2. $\sqrt{3x}=6$

3. $\sqrt{x+3}=x+1$

4. $\sqrt{(15x+10)}=2x+3$

A-REI.2

Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI.2

Claim #3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

- Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A-REI.A

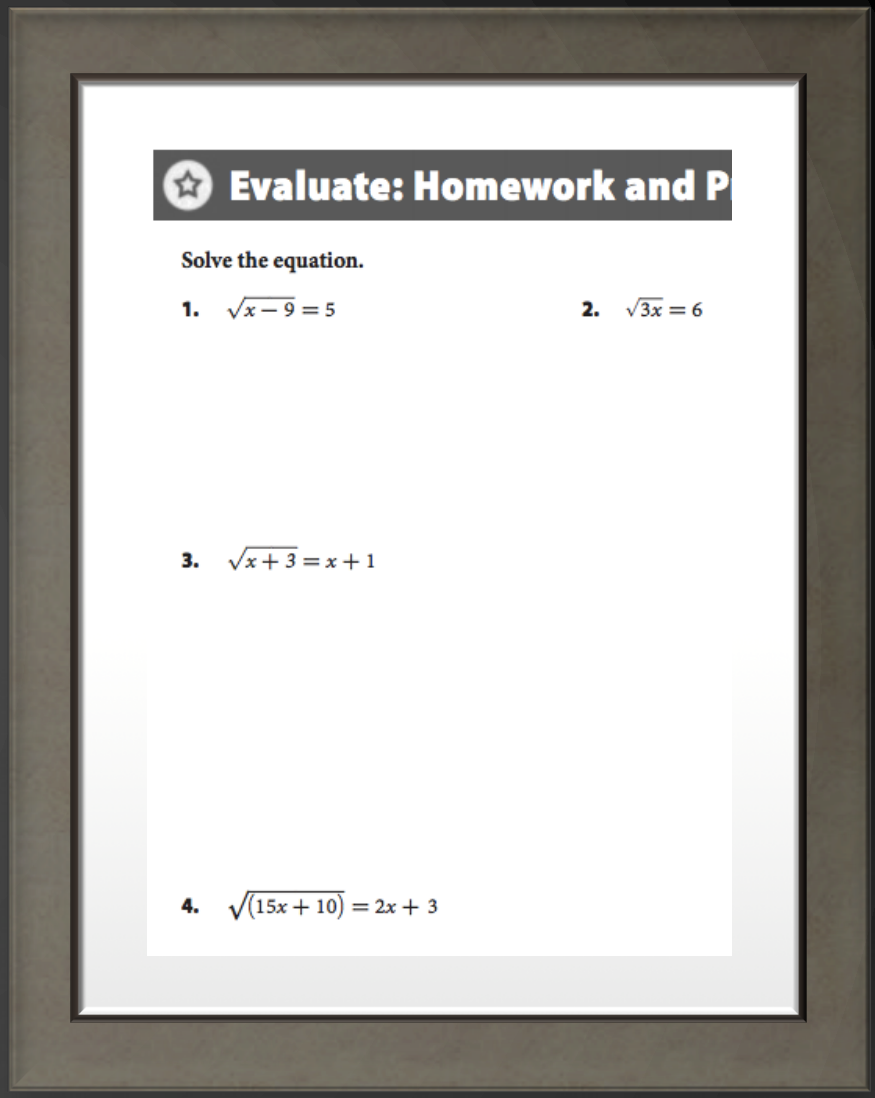
Claim #4: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

- **Understand solving equations as a process of reasoning and explain the reasoning.** [Master linear; learn as general principle.]

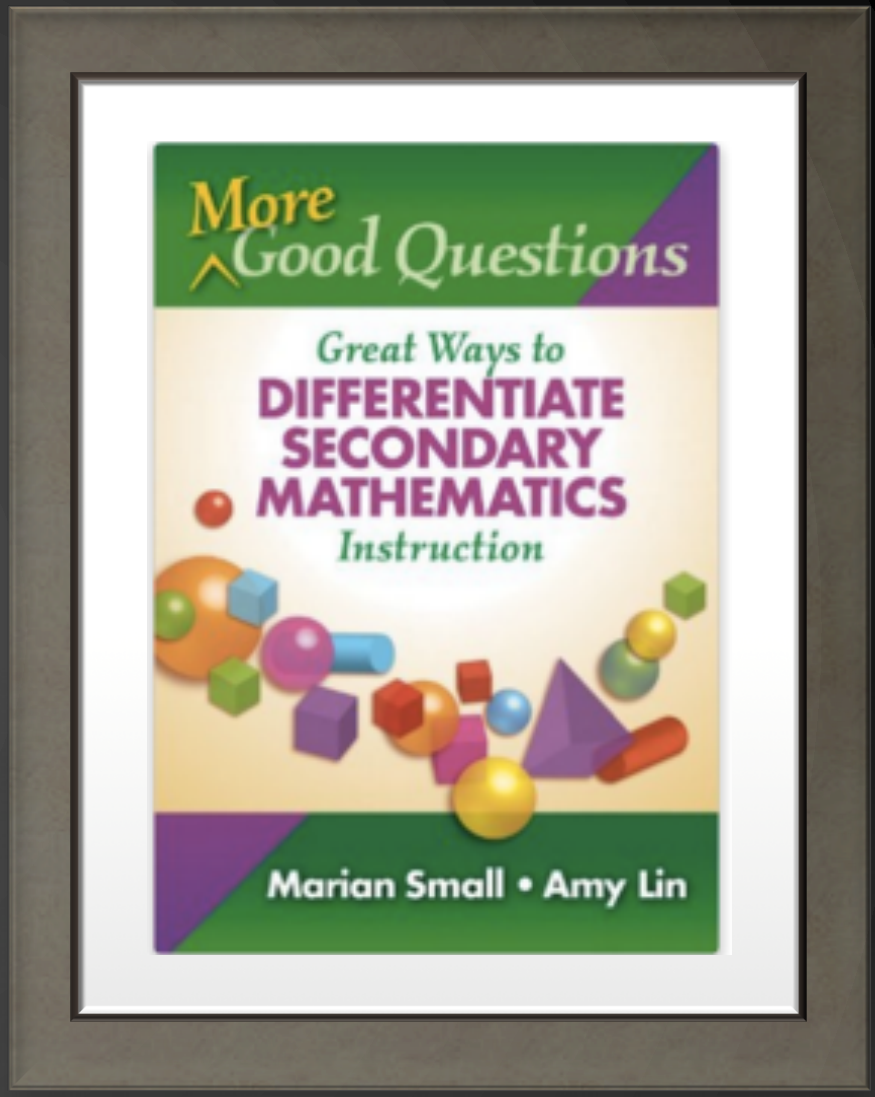
HOW CAN WE INCORPORATE THE SMPS IN A LESSON?

SMP #3: Construct viable arguments and critique the reasoning of others.

SMP #8: Look for and express regularity in repeated reasoning.



CREATING OPEN QUESTIONS



WAYS TO CREATE OPEN QUESTIONS

- Turn around a question
- Ask for similarities and differences
- Replace a number, shape, measurement unit and so forth with a blank
- Ask for a number sentence

TURNING AROUND A QUESTION

Give the answer and have students ask the question

- Example:
  - What is the hypotenuse of a right triangle if the legs are 3 units and 4 units long?
- Becomes:
  - One side of a right triangle is 5 units long. What could the other lengths be?

G-SRT. 8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. ★

G-SRT.C

Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

- **Define trigonometric ratios and solve problems involving right triangles.**

One side of a right triangle is 5 units long. What could the other lengths be?

What SMPs are being used?

- SMP #1: Make sense of problems and persevere in solving them.
- SMP #3: Construct viable arguments and critique the reasoning of others.
- SMP #5: Use appropriate tools strategically.
- SMP #6: Attend to precision.
- SMP #8: Look for and express regularity in repeated reasoning.

ASKING FOR SIMILARITIES AND DIFFERENCES

Choose two items- numbers, shapes, graphs, measurements, etc. and ask how they are alike and different.

- How is a circle and a sphere alike? How are they different?

G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★

G-GMD.3

Claim #4: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

- Use volume formulas for cylinders, pyramids, cones, and spheres to **solve problems**. ★

How is a circle and a sphere alike? How are they different?

What SMPs are being used?

- SMP #1: Make sense of problems and persevere in solving them.
- SMP #2: Reason abstractly and quantitatively.
- SMP #3: Construct viable arguments and critique the reasoning of others.
- SMP #6: Attend to precision.
- SMP #7: Look for and make use of structure.

REPLACING A NUMBER WITH A BLANK

Replace a number with a blank and allow students to choose the numbers to use.

- Example:
- $\frac{4m}{5} - \frac{1}{2} = -\frac{25}{2}$, solve for *m*
- Becomes:
  - The solution to an equation is *m* = -15 and involves a fraction. What might that equation be?

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-REI.1

Claim #3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

- Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.A

Claim #4: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

- **Understand solving equations as a process of reasoning and explain the reasoning.** [Master linear; learn as general principle.]

The solution to an equation is $m = -15$ and involves a fraction. What might that equation be?

What SMPs are being used?

- SMP #1: Make sense of problems and persevere in solving them.
- SMP #2: Reason abstractly and quantitatively.
- SMP #3: Construct viable arguments and critique the reasoning of others.
- SMP #6: Attend to precision.
- SMP #8: Look for and express regularity in repeated reasoning.

ASKING FOR A SENTENCE

Students can be asked to create a sentence that includes certain words or numbers.

- Example:
- Create a sentence that includes the words “linear” and “increasing” and the numbers 4 and 9.
- Possible answers:
  - *An increasing linear pattern could include the numbers 4 and 9.*
  - *In a linear pattern starting at 4 and increasing by 9, the tenth number will be 85.*
  - *A linear pattern that is increasing by 9 grows faster than one that is increasing by 4.*

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★

F-IF.B

Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

- **Interpret functions that arise in applications in terms of the context.** [Linear, exponential, and quadratic]

F-IF.B

Claim #4: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

- **Interpret functions that arise in applications in terms of the context.** [Linear, exponential, and quadratic]

Create a sentence that includes the words “linear” and “increasing” and the numbers 4 and 9.

What SMPs are being used?

- SMP #1: Make sense of problems and persevere in solving them.
- SMP #2: Reason abstractly and quantitatively.
- SMP #3: Construct viable arguments and critique the reasoning of others.
- SMP #4: Model with mathematics.
- SMP #6: Attend to precision.
- SMP #8: Look for and express regularity in repeated reasoning.

WHICH OF THESE FOUR RELATIONSHIPS ARE MOST ALIKE? WHY?

- $y = 4x + 5$
- $y = 3x + 5$
- $y = 4x + 3$
- $y = 4x - 3$

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* ★

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F-IF.B

Claim #4: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

- **Interpret functions that arise in applications in terms of the context.** [Linear, exponential, and quadratic]

- Draw a graph of a parabola that grows quickly and the graph of a parabola that grows slowly. What are the equations of these parabolas?

- A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

A-CED.A

Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

- **Create equations that describe numbers or relationships.** [Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only]

A-CED.A

Claim #4: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

- **Create equations that describe numbers or relationships.** [Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only]

- There are close to 3,000 seats in a theater. There are 20 rows, and the numbers of seats in successive rows form an arithmetic sequence. How many seats can be in the front row?

- F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. ★

F-BF.A

Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

- **Build a function that models a relationship between two quantities.** [For F.BF.1, 2, linear, exponential, and quadratic]

F-BF.A

Claim #4: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

- **Build a function that models a relationship between two quantities.** [For F.BF.1, 2, linear, exponential, and quadratic]

- A quadratic equation of the form $x^2+10x + c$ can be factored in the form $(x+a)(x+b)$, where a and b are whole numbers.
- Another quadratic equation of the form $x^2+12x+d$ can be factored in the form $(x+e)(x+f)$, where e and f are whole numbers.
- Which do you think is greater: c or d? Why?

- A-SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines. ★

A-SSE.B

Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

- **Write expressions in equivalent forms to solve problems.** [Quadratic and exponential]

A-SSE.B

Claim #4: Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

- **Write expressions in equivalent forms to solve problems.** [Quadratic and exponential]

- Two of the vertices of a quadrilateral are (1, 3) and (5, 3). The other vertices are also in Quadrant 1. What kind of quadrilateral could it be?

- G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-CO.A

Claim #3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

- **Experiment with transformations in the plane.**

HOW CAN WE REMAKE THIS INTO AN OPEN QUESTION?

- Write the equation of the line in slope-intercept form for: Slope is 3, and (1, 5) is on the line.

- A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A-REI.D

Claim #2: Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem-solving strategies.

- **Represent and solve equations and inequalities graphically.** [Linear and exponential; learn as general principle.]

A-REI.10

Claim #3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

- Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

THANK YOU!